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ABSTRACT

The task analysis for Developing Mathematical Processes, Arithmetic, Book 2: Writing Mathematical Sentences is presented. The major objective of this text is to have children write and validate mathematical sentences which represent the results of the compare-and-equalize process. The major components of the task analysis of Book 2 are Writing Mathematical Sentences and Validating Mathematical Sentences. The complete task analysis is presented as a list of 53 objectives. (JG)

Working Paper No. 49

The Task Analysis for Developing Mathematical Processes, Arithmetic Book 2: Writing Mathematical Sentences

Report from the Project on Individually Guided Elementary Mathematics, Phase 2: Analysis Of Mathematics Instruction

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The Task Analysis for Developing Mathematical Processes, Arithmetic, Book 2: Writing Mathematical Sentences

by

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Report from the Project on

Individually Guided Elementary Mathematics

Phase Two, Analysis of Mathematics Instruction

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The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Working Paper is from the Project on Individually Guided Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project has developed and tested a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Analysis of Mathematics Instruction is currently the only active phase of the mathematics project and has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focused on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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ABSTRACT

This paper presents the task analysis for Developing Mathematical Processes, Arithmetic, Book 2: Writing Mathematical Sentences.

I

INTRODUCTION

The purpose of this paper is to present the completed task analysis for Arithmetic, Book 2: Writing Mathematical Sentences of the Developing Mathematical Processes (DMP) series being prepared by the Analysis of Mathematics Instruction project of the University of Wisconsin Research and Development Center for Cognitive Learning. The identification of content, the task analysis, and the organization of behaviors identified through task analysis into topics constitute the first steps in the development sequence (Harvey, Romberg, and Fletcher, 1969).

The initial objective of the DMP program is for students to learn to accurately complete equations of the general form $A = B \pm X$. Conceptually, these equations or mathematical statements simply require the students to compare two objects with respect to a metrizable property and to make them equivalent on that property by adding some amount to or taking some amount from one of the objects. This compare-and-equalize process is considered to be a fundamental part of basic mathematics and is well within the intellectual capabilities of young children (Romberg and Roweton, 1969; Romberg and Gornowicz, 1970; and Romberg and Planert, 1970).

The main objective of Arithmetic, Book 1: Comparing and Equalizing Objects and Sets is to have children equalize objects and sets since this is a prerequisite behavior to using numbers in forming mathematical sentences. In Arithmetic, Book 2, the objective is to have children write and validate mathematical sentences which represent the results of the compare-and-equalize process.

II

THE TASK ANALYSIS

Following specification of the mathematical goal of writing and validating mathematical sentences, a series of steps followed which identified the behaviors needed to reach this objective. The specification of these behavioral objectives and their arrangement into prerequisite skeins is accomplished by a process known as task analysis. Here each unit or concept is analyzed in terms of its subconcepts, properties, or attributes together with the rules necessary for their combination as well as prerequisite behaviors the student must possess for any unit. These prerequisite behaviors are then used to develop a chart relating the units.

The task analysis provides direction for the staff of the Analysis of Mathematics Instruction Project in sequencing the concepts but this is only one of its contributions to the development effort. It helps the team to describe general problem-solving processes for mathematics. Another contribution is that, since the task analysis is described in terms of student behaviors, it is a complete guide for the generation of valid test items and reliable tests which are used in the evaluation of the curriculum being developed. Finally, the task analysis helps to identify connections between the various subject matter areas.

The major components of the task analysis of Arithmetic, Book 2, can be described in terms of three areas. (See Figure 1.) The initial component, "Task Analysis for Arithmetic, Book 1", includes the prerequisite behaviors previously identified in the task analysis for Book 1 (Romberg, Harvey, and McLeod, 1970). Many behaviors specified

in that task analysis are repeated here to indicate the inherent relationship between these analyses. The second component, "Writing Mathematical Sentences", includes the behaviors needed to produce sentences. And in the last component, "Validating Mathematical Sentences", the behaviors needed to establish the empirical truthfulness of sentences are identified.

Major Components of the Task Analysis for
Developing Mathematical Processes, Arithmetic, Book 2:
Writing Mathematical Sentences

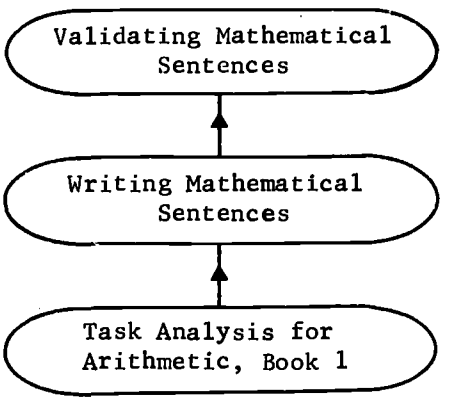
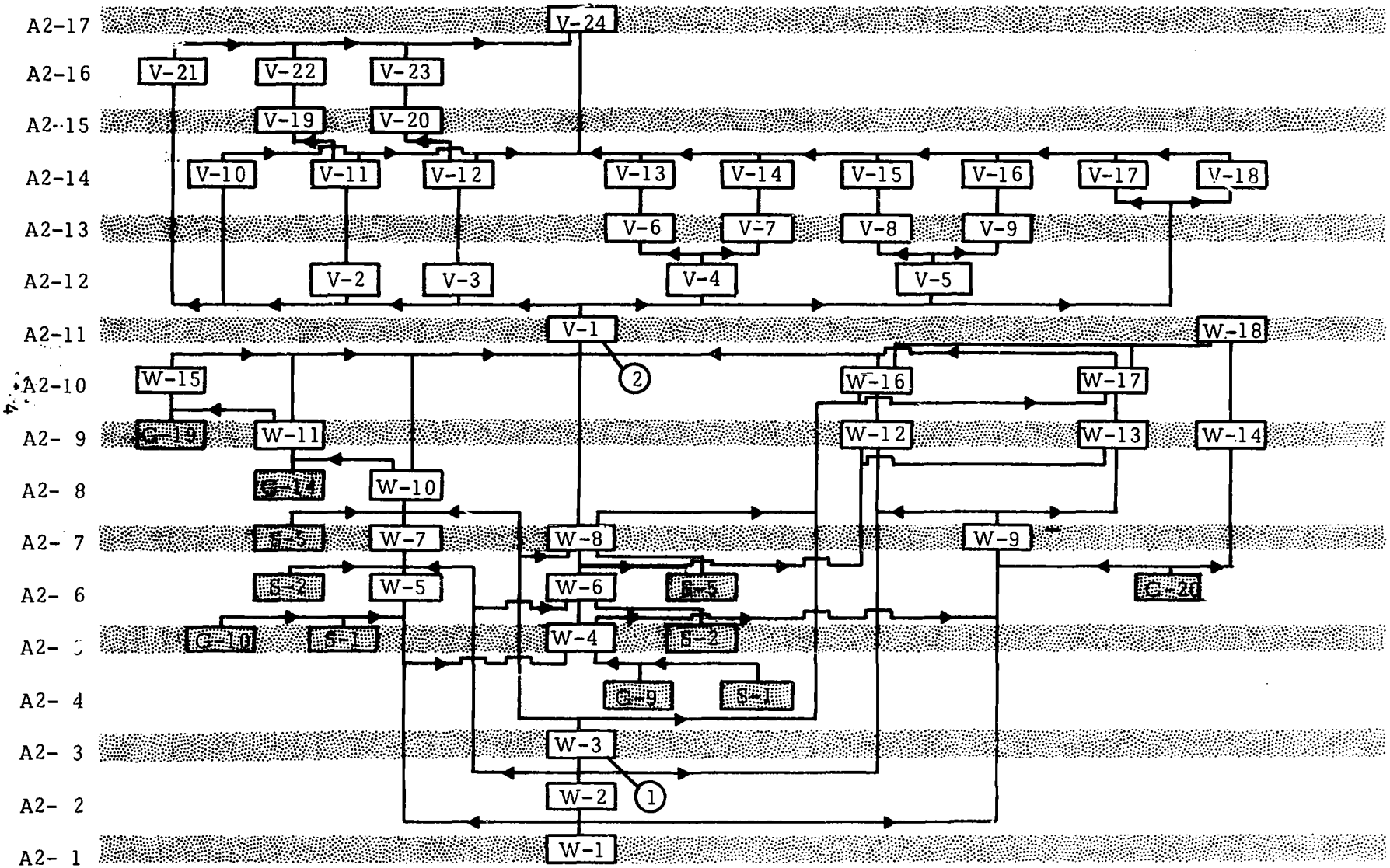


Figure 1

Figure 2 represents the task analysis for Arithmetic, Book 2: Writing Mathematical Sentences. The figure includes behaviors at 17

Figure 2

Task Analysis for Developing Mathematical Processes, Arithmetic,
Book 2: Writing Mathematical Sentences



different levels. Behavioral objectives are indicated in terms of boxes (▭); these objectives are listed in Table 1. Circles (○) designate related behaviors from other task analyses; names for these analyses are listed in Table 2. Moving upward on the chart indicates a progression from subordinate to superordinate behaviors. The lines show the relationship between behaviors and the arrowheads (→) indicate the direction of dependence.

Table 1 contains the specific behavioral objectives of the task analysis presented in Figure 2. Each behavioral objective is labeled with a two-part code. The first objective, for example, is A2-1, W-1. The first part of the code, A2-1, indicates that this objective is found in the task analysis of Arithmetic Book 2 on the first level; the second part of the code, W-1, corresponds to the label on the box that shows the location of this objective in Figure 2. In the label W-1, the letter refers to a category of objectives and the number indexes the objectives within the category. Here the letter W indicates that the objective is related to writing sentences, while the letter V designates objectives that are used in validating sentences.

The task analysis presented in Figure 2 contains some objectives that are repeated from the task analysis for Arithmetic, Book 1 and are shaded in this figure. As Table 1 indicates, the complete statement of these repeated objectives is found in the Working Paper entitled "The Task Analysis for Developing Mathematical Processes, Arithmetic, Book 1: Comparing and Equalizing Objects and Sets" (Romberg, Harvey, and McLeod, 1970).

[Text continued on Page 12.]

Table 1

Behavioral Objectives for the Task Analysis of
Developing Mathematical Processes, Arithmetic, Book 2:
Writing Mathematical Sentences

Level	Label	Objective
A2-1	W-1	When shown one or more of the connectives =, ≠, <, >, +, and -, identifies a specified connective.
A2-2	W-2	When shown one of the connectives =, ≠, <, >, +, and -, attaches the correct verbal label to it.
A2-3	W-3	Given a description of one of the connectives =, ≠, <, >, +, and -, correctly writes the specified connective.
A2-4	G-9	Working Paper No. 48, Table 1 (A1-14, G-9).
A2-4	S-1	Working Paper No. 48, Table 1 (A1-6, S-1)
A2-5	G-10	Working Paper No. 48, Table 1 (A1-15, G-10)
A2-5	S-1	Working Paper No. 48, Table 1 (A1-6, S-1)
A2-5	W-4	Given objects A and B having number measures a and b, respectively, correctly identifies the sentences $a = a$ and $a \neq b$.
A2-5	S-2	Working Paper No. 48, Table 1 (A1-7, S-2)
A2-6	S-2	Working Paper No. 48, Table 1 (A1-7, S-2)
A2-6	W-5	Given objects A and B having number measures a and b, respectively, correctly identifies the sentences $a < b$ and $b > a$.

Table 1 (continued)

Level	Label	Objective
A2-6	W-6	Given objects A and B having number measures a and b, respectively, and given the sentences $a = a$ and $a \neq b$, correctly reads the sentences.
A2-6	S-5	Working Paper No. 48, Table 1 (A1-9, S-5).
A2-6	G-20	Working Paper No. 48, Table 1 (A1-18, G-20).
A2-7	S-5	Working Paper No. 48, Table 1 (A1-9, S-5).
A2-7	W-7	Given objects A and B having number measures a and b, respectively, and given the sentences $a < b$ and $b > a$, correctly reads the sentences.
A2-7	W-8	Given objects A and B having number measures a and b, respectively, correctly writes the sentences $a = a$ and $a \neq b$.
A2-7	W-9	Given objects A, B, and C having number measures a, b, and c, respectively, correctly identifies the sentences $a = b + c$ and $b = a - c$.
A2-8	G-14	Working Paper No. 48, Table 1 (A1-17, G-14).
A2-8	W-10	Given objects A and B having number measures a and b, respectively, correctly writes the sentences $a < b$ and $b > a$.
A2-9	G-19	Working Paper No. 48, Table 1 (A1-18, G-19).
A2-9	W-11	Given objects A, B, and C having number measures a, b, and c, respectively, where the three objects have been placed in order from smallest to largest, writes the sentences $a < b < c$ and $c > b > a$.

Table 1 (continued)

Level	Label	Objective
A2-9	W-12	Given objects A, B, and C having number measures a, b, and c, respectively, and given the sentence $a = b + c$, correctly reads the sentence.
A2-9	W-13	Given objects A, B, and C having number measures a, b, and c, respectively, and given the sentence $b = a - c$, correctly reads the sentence.
A2-9	W-14	Given objects A, B, and C having number measures a, b, and c, respectively, correctly states both the sentences $a = b + c$ and $b = a - c$.
A2-10	W-15	Given objects A, B, and C having number measures a, b, and c, respectively, such that $a < b$ and $b < c$, writes $a < c$.
A2-10	W-16	Given objects A, B, and C having number measures a, b, and c, respectively, correctly writes the sentence $a = b + c$.
A2-10	W-17	Given objects A, B, and C having number measures a, b, and c, respectively, correctly writes the sentence $b = a - c$.
A2-11	V-1	Chooses a model to apply to a given mathematical sentence: the identity model, the pictorial model, or the physical model.
A2-11	W-18	Given objects A, B, and C having number measures a, b, and c, respectively, correctly writes both the sentences $a = b + c$ and $b = a - c$.

Table 1 (continued)

Level	Label	Objective
A2-12	V-2	Given a mathematical sentence involving the numbers a, b, (and c), constructs pictorial models of a, b, (and c).
A2-12	V-3	Given a mathematical sentence involving the numbers a, b, (and c), constructs physical models of a, b, (and c).
A2-12	V-4	Given a mathematical sentence involving the numbers a, b, and c, constructs physical models of a, b, and c.
A2-12	V-5	Given a mathematical sentence involving the numbers a, b, and c, constructs pictorial models of a, b, and c.
A2-13	V-6	Given numbers b and c and physical models of b and c, constructs a physical model of $b + c$.
A2-13	V-7	Given numbers a and c and physical models of a and c, constructs a physical model of $a - c$.
A2-13	V-8	Given numbers b and c and pictorial models of b and c, constructs a pictorial model of $b + c$.
A2-13	V-9	Given numbers a and c and pictorial models of a and c, constructs a pictorial model of $a - c$.
A2-14	V-10	Given a complete list of previously validated results, searches for the (identity model of the) sentence $a = a$, $a \neq b$, $a < b$ and $b > a$, or $a < b < c$ and $c > b > a$ in the list.

Table 1 (continued)

Level	Label	Objective
A2-14	V-11	Given a mathematical sentence $a = a$, $a \neq b$, $a < b$ and $b > a$, or $a < b < c$ and $c > b > a$, and given pictorial models of the numbers a, b, and c, constructs a pictorial model of the sentence.
A2-14	V-12	Given a mathematical sentence $a = a$, $a \neq b$, $a < b$ and $b > a$, or $a < b < c$ and $c > b > a$, and given physical models of the numbers a, b, and c, constructs a physical model of the sentence.
A2-14	V-13	Given the mathematical sentence $a = b + c$ and physical models of the numbers a, b, and c, constructs a physical model of the sentence $a = b + c$.
A2-14	V-14	Given the mathematical sentence $b = a - c$ and physical models of the numbers a, b, and c, constructs a physical model of the sentence $b = a - c$.
A2-14	V-15	Given the mathematical sentence $a = b + c$ and pictorial models of the numbers a, b, and c, constructs a pictorial model of the sentence $a = b + c$.
A2-14	V-16	Given the mathematical sentence $b = a - c$ and pictorial models of the numbers a, b, and c, constructs a pictorial model of the sentence $b = a - c$.
A2-14	V-17	Given a complete list of previously validated results, searches for the (identity model of the) sentence $a = b + c$ in the list.

Table 1 (continued)

Level	Label	Objective
A2-14	V-18	Given a complete list of previously validated results, searches for the (identity model of the) sentence $b = a - c$ in the list.
A2-15	V-19	Given the mathematical sentence "If $a < b$ and $b < c$, then $a < c$," and pictorial models of a, b, and c, constructs pictorial models of $a < b$ and $b < c$.
A2-15	V-20	Given the mathematical sentence "If $a < b$ and $b < c$, then $a < c$," and physical models of a, b, and c, constructs physical models of $a < b$ and $b < c$.
A2-16	V-21	Given a complete list of previously validated results, searches for the (identity model of the) sentence "If $a < b$ and $b < c$, then $a < c$" in the list.
A2-16	V-22	Given the numbers a, b, and c, and pictorial models of $a < b$ and $b < c$, constructs a pictorial model of $a < c$.
A2-16	V-23	Given the numbers a, b, and c, and physical models of $a < b$ and $b < c$, constructs a physical model of $a < c$.
A2-17	V-24	Given a mathematical sentence and a model of the sentence, describes whether the sentence is true or false on the basis of the given model.

Table 2 gives the titles of the related task analyses that are represented by the circles in Figure 2. The titles are numbered so that they correspond to the numbered circles that represent these related task analyses in the diagram.

Table 2

Related Task Analyses

1 ORIENTATION

2 MODELING NUMERALS

A task analysis, however, does not indicate how instruction will take place. Since instruction must proceed sequentially, decisions have been made as to which objectives are to be taught in what order. The sequence of instruction chosen for Arithmetic, Book 2, is indicated by a topic outline (Table 3).

Arithmetic, Book 2, after reviewing the compare-and-equalize process, proceeds to forming comparison sentences with symbols, to writing comparison sentences, to writing equalization sentences, and finally, to constructing and using an equalization table. It should be pointed out that the validation objectives are brought into this sequence whenever the student constructs sentences. In validating a sentence the student relates the abstract sentence to a physical, pictorial, or (previously validated) symbolic model in order to decide whether the sentence is true or false.

Table 3

Topic Outline for Developing Mathematical Processes, Arithmetic, Book 2: Writing Mathematical Sentences

Topic	
1	Comparing two objects or sets, a review.
2	Comparing two objects or sets by counting.
3	Comparing two objects or sets using the symbols 0-10, =, and ≠.
4	Writing the numerals 0-10.
5	Writing comparison sentences using the numerals 0-10, =, ≠.
6	Comparing numerals representing objects and sets.
7	Writing the equalization sentence a = b + c.
8	Writing the equalization sentence a = b - c.
9	Writing mathematical sentences.
10	Constructing and using an equalization table.

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